Journal of **N**ovel **R**esearches on **S**mart **P**ower **S**ystems - Vol. 14 - No. 1 – Spring 2025
P-ISSN: 2322-2468, E-ISSN: 2981-0272, https://jeps.dezful.iau.ir/

Research Article

Investigation of Stability and Performance of the Extended Hyperchaotic Chen System

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Abstract:
Chaotic and hyperchaotic dynamical systems have emerged as powerful tools in modern engineering, particularly in the domains of secure communication, cryptography, and complex signal generation. Their hallmark characteristic—extreme sensitivity to initial conditions—renders their long-term behavior inherently unpredictable, thereby providing a natural source of entropy and randomness for information security applications. However, conventional chaotic systems, such as the Lorenz or original Chen systems, often possess only a single positive Lyapunov exponent, limiting their dynamic complexity and, consequently, their cryptographic strength. To address this limitation, recent research has focused on developing **hyperchaotic systems**—those with two or more positive Lyapunov exponents—which exhibit richer, higher-dimensional dynamics and are far more resistant to prediction and unauthorized decoding.

This paper introduces and rigorously analyzes a **novel extended hyperchaotic Chen system**, derived through strategic augmentation of the classical hyperchaotic Chen framework. The primary objective is to enhance the system's dynamic complexity while preserving its structural integrity and mathematical tractability, thereby creating a more robust platform for high-security communication protocols. The proposed system is defined by a set of four coupled, nonlinear, ordinary differential equations that incorporate additional state-dependent coupling terms and expanded parameter space, resulting in a more intricate phase-space trajectory compared to its predecessors.

The paper begins by formally defining the mathematical structure of the extended system. Starting from the well-known Chen system:

$$\dot{x} = a(y - x)$$
$$\dot{y} = (c - a)x - xz + cy$$
$$\dot{z} = xy - bz$$

the authors extend it into a four-dimensional hyperchaotic form and further generalize it to the proposed extended version, which includes additional nonlinear interaction terms among the state variables. This extension is not arbitrary; it is designed to increase the system's degrees of freedom and to introduce new pathways for energy exchange among its internal states, thereby amplifying its chaotic behavior.

The core of the paper lies in the **comprehensive stability and complexity analysis** of the proposed system. To quantitatively verify its hyperchaotic nature, the full spectrum of **Lyapunov exponents** is computed using established numerical algorithms. The results are striking: the system exhibits **two positive Lyapunov exponents** (e.g., $\lambda 1$=0.2832 , $\lambda 2$=0.0541), one near-zero exponent ($\lambda 3 \approx 0$), and one negative exponent ($\lambda 4 = -12.356$). The presence of two positive exponents is the definitive signature of hyperchaos, indicating that the system's trajectories diverge exponentially in two independent directions in its four-dimensional phase space. This is a significant advancement over the original Chen system, which is merely chaotic.

To further validate and illustrate the system's dynamic richness, extensive **numerical simulations** are conducted in the MATLAB environment. The simulations compare the proposed extended system with the standard Chen system under identical initial conditions and parameter settings. The results, presented through time-series plots, 2D and 3D phase portraits, and error-dynamics graphs, consistently demonstrate that the extended system exhibits:

- A **more complex and folded attractor structure** in phase space, indicative of higher information capacity.
- **Faster divergence of initially proximate trajectories**, as evidenced by the rapidly growing error between the states of the two systems.
- **Enhanced ergodicity and mixing properties**, which are critical for ensuring that the generated signals uniformly cover the entire state space, a desirable trait for cryptographic applications.

The practical implications of this enhanced complexity are profound. In secure communication, a chaotic signal is often used as a **carrier wave** to mask an information-bearing message. The receiver, equipped with a synchronized copy of the chaotic system, can then demask the signal to recover the original message. The security of this scheme hinges on the unpredictability of the carrier. A hyperchaotic carrier with multiple positive Lyapunov exponents is exponentially harder to reconstruct or predict than a simple chaotic one, as an eavesdropper would need to accurately estimate a far more complex and higher-dimensional dynamic model. Consequently, the proposed extended Chen system is posited as a superior candidate for use as a **robust and reliable carrier in telecommunications networks**.

Moreover, the paper implicitly addresses the critical issue of **synchronization**, which is a prerequisite for any chaotic communication scheme. While the full synchronization control design is beyond the scope of this work, the detailed stability analysis provides the foundational understanding necessary for future controller development. The identification of the system’s Lyapunov spectrum and its open-loop dynamics are essential first steps in designing adaptive or sliding-mode controllers that can achieve master-slave synchronization despite the system’s inherent instability.

In conclusion, this work makes a significant contribution to the field of nonlinear dynamical systems by proposing and validating a novel, extended hyperchaotic Chen system. Through rigorous mathematical analysis and comprehensive numerical simulation, the paper demonstrates that the proposed system possesses a higher degree of dynamic complexity than its classical counterpart, as confirmed by its dual positive Lyapunov exponents and its richer phase-space behavior. This enhanced complexity directly translates to a tangible improvement in its potential for practical applications, most notably in the realm of **high-security, chaos-based communication systems**. The results confirm the effectiveness of the extension methodology and establish the system’s viability as a next-generation platform for secure data transmission. Future work will focus on the practical implementation of this system in a hardware-in-the-loop communication setup and the development of a robust synchronization protocol to enable its real-world deployment as a secure carrier signal.

Keywords: Chaos, Stability, Extended Chen System, Hyperchaotic System.

Received: 27 Dec. 2024
Revised: 24 Mar. 2025
Accepted: 8 Apr. 2025

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Citation: H. Soheili Rad, E. Moradi “Investigation of Stability and Performance of the Extended Hyperchaotic Chen System”, Journal of Novel Researches on Smart Power Systems, vol. 14, no. 1, pp. 1-8, June 2025 (in Persian).